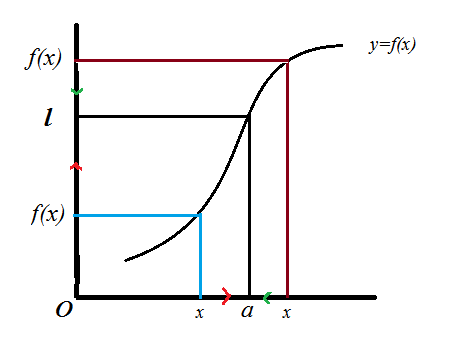
**Limits, Continuity & Differentiability**

***Introduction:*** In this chapter we will study about limit that is the core tool of calculus and all other calculus concepts are based on it. A function can be undefined at a point, but we can think about what the function "approaches" as it gets closer and closer to that point (this is the "limit"). Also the function may be defined at a point, but it may approach a different limit. There are many, many times where the functional value is the same as the limit at a point. Limit is used to define continuity, derivative and integral of a function.

***Limit of a function****:* The number is called limit of a function at a point if approaches closer and closer to from both sides and consequently approaches closer and closer to Symbolically it is written as,

*****Graphical representation of “limit of a function” at a point:***

***Mathematical* or  *definition of limit of a function:*** The number is called limit of a function at approaches if for any given positive number , we can find another positive number such that , for all values of *x* satisfying

.

Symbolically it is written as,

**Left Hand Limit:** If the values of can be made as close as we like to by taking values of *x* sufficiently close to (but less than) then we write,

**Right Hand Limit:** If the values of can be made as close as we like to by taking values of *x* sufficiently close to (but greater than) then we write,

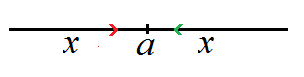
***Existence of limit of a function at :***

The limit of a function at that isexists if

1. *.*

***Fundamental Properties of limit:***

If are two functions and

***Change of limit of a variable*:**

**Left hand limit:**

Let and when.

Now, .

**Right hand limit:**

Let and when.

Now, .

***Problem-01:*** A function  is defined as follows:



Does  exist ?

***Solution*:**Given that, 

Since . So  does not exist.

***Problem-02:*** A function  is defined as follows:



Find the value of .

***Solution*:**Given that, 



Since . So  exists.

The limiting value is,

.

***Problem-03:*** If  then find limits from the left and the right of . Does the limit of  at  exist?

***Solution*:**Given that, 

Here,  and  both are exist but they are not same.

i.e, . So  does not exist.

***Problem-04:*** If  then find limits from the left and the right of . Does the limit of  at  exist ?

***Solution*:**Given that, 

Here,  and  both are exist but they are not same. i.e, . So  does not exist.

***Problem-05:*** A function  is defined as follows:



Discuss the existence of .

***Solution*:**Given that, 



Here,  and  both are exist but they are not same.

i.e, . So  does not exist.

***Problem-06:*** A real function is defined by .

Find a).  ; b)  and c). .

***Solution*:**Given that, 

**1st part a:**  

  .

Here,  and  both are not exist. So  does not exist.

**2nd part b:**

Let  then 

Now, 







.

**3rd part c:**

Let  then 

Now, 







.

***Homework*:**

***Problem-01*:** A function  is defined as follows:



Find the value of .

***Problem-02*:** A function  is defined as follows:



Find the value of .

***Problem-03:*** If  then find limits from the left and the right of . Does the limit of  at  exist ?

***Problem-04:*** If  then find limits from the left and the right of . Does the limit of  at  exist ?

***Problem-05:*** If  then show that does not exist but exists.

***Problem-06:*** If  then find limits from the left and the right of . Does the limit of  at  exist?

***Some important limits:***

1.  **2.** 

**Proof:** Given that, **Proof:** Given that,

****

1.  **4.** 

**Proof:** Given that, **Proof:** Given that,



1. 

**Proof:** Given that,

****

**L’ Hospital’s Rule:** If two functions and are continuous at  , also their derivatives , are continuous at this point and  but then L’ Hospital’s rule states as,



In case, , the rule maybe extended.

**Indeterminate forms:** If  then it is called an indeterminate form at . The forms , , , ,  and  are also indeterminate forms.

**Evaluate the following limits:**

**Problem 01: Find**  **Problem 02: Find** 

**Sol:** Given that, **Sol:** Given that,

****

**Problem 03: Find**  **Problem 04: Find** 

**Sol:** Given that, **Sol:** Given that,

**Problem 05: Find**  **Problem 05: Find** 

**Sol:** Given that, **Sol:** Given that,

****

**Problem 06: Find**  **Problem 07: Find** 

**Sol:** Given that, **Sol:** Given that,

****

**Problem 08: Find**  **Problem 09: Find** 

**Sol:** Given that, **Sol:** Given that,

**Homework:**

**Problem 01: Find**  Ans: 1

**Problem 02: Find**  Ans: 

**Problem 03: Find**  Ans: 

**Problem 04: Find**  Ans: 

**Problem 05: Find**  Ans: 

**Problem 06: Find**  Ans: 1

**Continuity:** A function is said to be continuous at a point  provided the following three conditions are satisfied:

1.  exists,
2.  is defined,
3. .

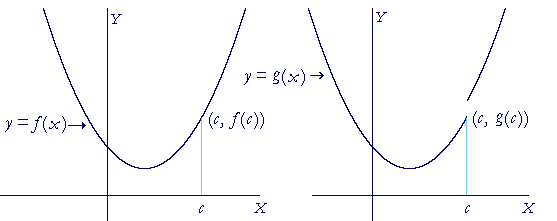
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Fig. (a) Continuous function. Fig. (b) Discontinuous function.

If one or more of the conditions of this definition fails to hold, then the function  is discontinuous at .

***Problem-01:*** A function  is defined as follows:



Discus the continuity at .

***Solution*:**Given that, 

Here, . So  exists and the limiting value is,

.

Now, the functional value at is,



Since, , the given function is continuous at .

***Problem-02:*** Test the continuity of the function  at the point 

***Solution*:**The given function is, 





Here, . So  exists and the limiting value is,

.

Now, the functional value at is,





Since, , the given function is continuous at .

***Problem-03:*** If for what value of *a* ,  is continuous at 

***Solution*:**Given that, 

And, the functional value at is,





Now, the given function will be continuous at ,

if 







 (*Ans*.)

***Problem-04:*** If then test the continuity at 

***Solution*:**Given that, 











.

And, the functional value at is,



Since, , the given function is continuous at .

***Problem-05:*** If then test the continuity at 

***Solution*:**Given that, 











.

And, the functional value at is,



Since, , the given function is continuous at .

***Problem-06:*** A function  is defined as follows:



Discus the continuity at .

***Solution*:**Given that, 

Here, . So  does not exist.

Hence, the given function is discontinuous at .

***Homework*:**

***Problem-01*:** A function  is defined as follows:



Test the continuity at .

***Problem-02*:** Discuss the continuity of the function  at the point 

***Problem-03:*** Test the continuity of the function  at the point 

***Problem-04:*** Find a non-zero value for the constant *k* that makes  continuous at.

***Problem-05:***  If then test the continuity at 

***Problem-06:***  If then test the continuity at 

***Problem-07:***  If then test the continuity at 

***Differentiability of a function***: The derivative of  with respect to *x* (for any particular value of ) is denoted by  or  and defined as,





Provided this limit exists.

***Existence of Derivative:***  A function  is called differentiable at  if the left hand derivative and right hand derivative at this point i.e,



and 

are both exist and equal.

**Problem 01:** A function  is defined as follows:



Discuss the differentiability at and .

**Solution:** Given that,



**1st Part:** For,

Since *R.H.D* does not exist. So the function is not differentiable at .

**2nd Part:** For ,

****

Since  does not exist. So the function is not differentiable at .

**Problem 02:** A function  is defined as follows:



Discuss the differentiability at and .

**Solution:** Given that,



**1st Part:** For,

Since  does not exist. So the function is not differentiable at .

** 2nd Part:** For ,

Since  exists. So the function is differentiable at .

**HOMEWORK:**

**Problem 01:** A function  is defined as follows:



Discuss the differentiability at .

**Problem 02:** Discuss the differentiability of the function  at the point and 

**Problem 03:** A function  is defined as follows:



Discuss the differentiability at and .

**Problem 04:** A function  is defined as follows:



Discuss the differentiability at and .

**Problem 05:** A function  is defined as follows:



Discuss the differentiability at .